

Game of Cheating

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Abstract. We develop a model of risky information transmission between a higher and a lower ability agent, which closely resembles cheating behavior. Without assuming any synergy effects between agents or correlation of outputs, we derive a substitution and a strategic effect, both of which dictate the behavior of agents when information transmission is feasible. We show that the higher ability agent may find it strategically useful to moderate her effort in order to limit the supply of information and curb her risk. We also find that the use of a relative compensation scheme may cause cheating activity to increase, when the coefficient of relative incentives is not sufficiently high. Moreover, even when relative incentives are strong enough to mitigate cheating, this seriously distorts the incentives causing high ability agents to further cut down on effort. We conclude that the performance of the group is higher when similar ability agents are matched together and that the only cost-free way to deter cheating is to match agents of similar abilities together as the risk of transmitting information will offset its benefit. The model is also extended to include the case where the higher ability agent can choose the level of her cheating supply.

Keywords: Cheating, absolute compensation, relative compensation, information transmission, strategic effects, cost of effort, type switching, performance incentives.

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1. Introduction

In 2012 around 70 Harvard students were disciplined for cheating during the final exam for the 'Introduction to Congress' class.¹ In Russia, the anonymous group 'Dissernet' revealed a number of plagiarism examples involving governors, federal lawmakers and other high-ranking officials.² The problem of academic cheating has become more acute during the last few decades. Sociological evidence suggests that pupils who get used to cheating at school, pursue dishonest behavior at colleges and universities and continue to act unethically later in their professional life.³ Moreover, cheating at school or university can contribute to corruption and different forms of dishonesty in adulthood.

From the perspective of economics, education serves a dual purpose. First, it improves the quality of human capital by increasing the productivity of workers. Second, it alleviates the adverse selection problem in the labor market by sending a clearer signal for the ability of a worker to potential employers.⁴ The problem of cheating defeats both purposes. Within a culture of cheating, educational achievement becomes a noisy signal discouraging higher ability workers from investing in human capital through education. As a result, investment in education becomes sub-optimal and the average productivity of the labor force remains under-developed.

The decisions of students regarding the effort they will exert for a class and the instructor's choice of evaluation method resembles several features of a problem of incentives. In the standard principal-agent model, the principal hires agents to produce output in a moral-hazard setting. Agents choose their optimal effort levels by maximizing the difference between expected compensation and costs of effort. As effort is usually unobservable, the principal can employ either an absolute or a relative scheme to compensate the agents based on their final performance. The compensation system is the principal's only instrument to incentivize the desired level of effort by the agents. Similarly, in the instructor-student setting, students also choose their optimal level regarding preparation for a course while incurring costs of effort. The instructor specifies the grading method in advance and observes only the performance of students on the controlled exams but not their levels of effort.

Nevertheless, there are several differences between the principal-agent model and the instructor-student relationship. First, the instructor may participate in the production process by supporting students in their efforts. In this context, the instructor's contribution can be viewed as an input or a common shock in the students' production functions. Second, the instructor is not a residual claimant of students' performance because giving lower

¹See Perez-Pena (2013).

²See Strauss (2014).

³See Lubimov (2016).

⁴See Spence (1973).

grades does not increase the instructor's payoff. Third, the instructor-student relationship is not always subject to a participation constraint. Finally, student performance is almost exclusively based on the level of information possessed by the student at the time of the test. Therefore, an information leak between students causes unlimited replication of performance without this to be accompanied with the appropriate increase in effort. If the information leak can be foreseen by the students, it will create substitution effects, making both high and low ability types adjust their effort choices in order to strike the optimal balance between disutility from effort and risk from getting caught while exchanging information.

In this paper we develop a formal agency framework analogous to the instructor-student relationship in order to investigate the effects of risky information transmission between agents. We consider two risk neutral agents (students) that interact, coordinate but also chose private strategies. Three important results of our model are outlined in the paper.

First, the model shows that in addition to the substitution of costly effort with risky cheating, the mere potential of cheating may create strategic effects. Specifically, in anticipation of a risky information exchange, a relatively higher ability student may want to moderate her effort in order to limit the available information to be transmitted to a lower ability student, and therefore cap the risk of getting caught on the act. In other words, the prospect of cheating alone discourages higher ability agents from performing at their potential.

Second, contrary to the popular belief that relative performance evaluation creates a cost to those who transmit information and therefore sufficiently mitigates cheating, we show that this is not always the case. A relative compensation scheme may cause cheating activity to increase, when the coefficient of relative incentives is not sufficiently high. Moreover, even when the relative performance coefficient becomes sufficiently high to deter cheating, this comes with serious welfare losses for the whole system. The latter result persists even when we generalize the model to allow agents to control the quantity of information they share.

Our third result concerns the optimal mix of agents with respect to their abilities in environments where cheating is possible. We show that, as the differential of abilities between agents increases, the high ability agent's effort increasingly deviates from the benchmark effort of no cheating. This is because, the higher the ability of an agent, the more information she will potentially be asked to supply, which results to a higher risk of being detected and punished. On the other hand, even though it is possible for the lower ability agent to move towards her benchmark effort in cases where availability of information is limited, cheating always has a negative impact on her effort because she partially substitutes effort with cheating. Therefore, we conclude that the educational output of the group is higher when similar ability agents are matched together. In fact, we show that the only truly cost-free

way to deter cheating is to match agents with similar abilities, so that the risk of transmitting information will overshadow the benefit.

It is useful to point out that our results are derived without assuming any synergy effects in the production functions of the agents; neither we allow for covariance of agents' outputs. Instead, all results depend solely upon the strategic behavior of agents regarding the horizontal externalities, which emerge from the asymmetrical risks and benefits from cheating for the two agents.

The paper is organized as follows. A review of the relevant literature is presented in section 2. Section 3 outlines the model. Section 4 describes the choice of agents in each possible state of information availability. Section 5 derives the equilibrium. Section 6 analyzes the comparative statics. Section 7 discusses the optimal matching of agents, section 8 contains a type switching extension of the baseline model and section 9 concludes.

2. Literature review

Arrow's (1963) seminal work on uncertainty and its welfare implications paved the way for numerous works that examined the influence of asymmetric information on market equilibria. Mirrlees (1975) was among the pioneers of the formal agency model. He considered the process of incentivizing agents for cost reducing effort in a moral hazard setting. The stream of literature that explores the properties of relative performance evaluation in contracts was initiated by Lazear and Rosen (1981), who contrasted a piece rate contract and a rank-order tournament. They showed that the optimal prizes in the rank-order tournament allow for efficient allocation of effort and outperform the linear piece-rate contract provided that the variance of the common shock is relatively higher than that of the idiosyncratic shock. Lazear and Rosen interestingly pointed out that the relative method allows for insurance trading between the principal and the agents making the agents more tolerant to stricter incentive schemes.

Nalebuff and Stiglitz (1981) and Holmstrom (1982) built a concrete theory on Lazear's and Rosen's foundations. Nalebuff and Stiglitz examined the optimality of relative performance evaluation through contests. They showed that for sufficiently large number of agents, optimal contest can provide the first-best solution of the perfect information benchmark. Also, penalizing the lowest ranked individual boosts effort more than rewarding the highest ranked individual. Holmstrom studied the conditions under which the absolute evaluation scheme is outperformed by the relative one. Optimal compensation of an agent under the relative performance scheme is shown to depend only on the output produced by the agent and the mean of outputs produced by other agents irrespective of the nature of idiosyncratic and common shocks. This is because the mean of outputs is a sufficient statistic for the common shock. Holmstrom concludes that costs arising from the moral hazard prob-

lem can be reduced by offering employees compensation based on their relative performance. Green and Stokey (1983) extended the theory of relative compensation assuming additional production settings.

The question regarding how rewards influence decision to pursue dishonest behavior gained much attention in the 'economics of criminal behavior' literature. One of the first works that analyzed criminal behavior from economic point of view was by Chadwick (1829), who asserts that prevention of the crime, rather than punishment for it, should become the main goal of the police as it makes the dishonest behavior cost-ineffective. Becker (1968) formally examines incentives to exhibit dishonest behavior. He concludes that the high rewards associated with criminality incentivize such behavior. Moreover, a material penalty is the most effective way to fight crime because it not only dis-incentivizes the perpetrator but can also restore the initial allocation of resources. Stigler (1970) considers the relative benefits of law enforcement. He notes that the penalty as an instrument for punishment can be explained by the nature of the supply of offenses. As a step further, Ehrlich (1973) develops a model for the offender's optimal choice of resource allocation under uncertainty that allows honest and dishonest behavior to not be mutually exclusive options and empirically tests it.

On the empirical side of the literature, Bunn, Caudill and Gropper (1992) draw the analogy between the crime of theft and cheating in the classroom. The benefit of committing a crime is equal to the value of the stolen good, whereas the expected cost depends on the probability of being caught and the severity of punishment. They point out two differences between the act of theft and cheating. First, a professor can affect the costs of dishonest behavior much more effectively than a police officer can. Costs of cheating in the classroom are higher than that of theft as the probability of being caught is higher and there can be several versions of the exam that make cheating more difficult. Second, if one student copies from another, the latter is not deprived from the answers, unlike the victim of a theft. Using Becker's framework for a logit model estimation, they establish that propensity to cheat negatively depends on GPA and positively depends on the perception of number of students who cheat. It is also affected by whether students have seen classmates cheating, cheaters being caught, the perceived severity of punishment and the students' attitude towards cheating. Surprisingly, the penalty for cheating was found to not be statistically significant. Kerkvliet (1994) uses a randomized response survey to test the results of Bunn, Caudill and Gropper. Mixon (1996) extends the logit model by Bunn, Caudill and Gropper distinguishing "serial cheaters" from "opportunity cheaters". Mixon successfully replicates the findings of Bunn et al. except for the penalty for cheating, which is found to be significant. On the theoretical side of the importance of the severity of punishment, Benabou and Tirole (2003) develop a model showing that increased punishment is treated by agents as a signal of distrust from

the principal and induces the former to exert lower level of effort.

A number of empirical studies examines how often people of different backgrounds from children to CEOs of big companies cheat and what is their motivation to pursue dishonest behavior. Evidence on frequency of cheating in most cases indicates an increasing tendency. Nevertheless, many of the studies examining this issue are not directly comparable as they employ different methodology to detect and compare cheating over the years. Bowers (1964) studied cheating in colleges using a sample of 5000 students at least half of which reported cheating. Crown and Spiller (1998) review literature on collegiate cheating. Among the main factors influencing the decision to cheat, they name gender, age, religion, ability (measured by grades and GPA), type and years of education, personality variables such as neuroticism and extroversion, situational factors such as honor codes, sanctions, etc. The results of most studies are consistent with those of McCabe and Trevino (1997). McCabe, Trevino and Butterfield (2001) conduct a meta-study on papers about cheating for one decade and point out that both individual and contextual factors affect dishonest behavior. They also note that cheating sharply increased during the last 30 years.

Magnus, Polterovich, Danilov and Savvateev (2002) compare students' attitude towards cheating in Russia, the United States, the Netherlands and Israel. They perform a survey that examines students' attitude towards their colleagues of type A who provide cheating, towards type B who cheat by copying answers and type C who detect cheating and inform a class teacher about it. Russian students seemed to dislike type C students more as compared to other countries. The Netherlands and the US were found to be less lenient towards students of type A and B. Stark variation in the results across countries is explained by differences in the coordination effect, which is influenced by cultural factors and the organizational features of the various educational systems.

Gallant and Drinan (2006) acknowledge that numerous aspects contribute to academic cheating, starting from psychological factors and extending to institutional effects. They suggest applying the organizational theories of Bolman and Deal (1997) and Huntington (1968) for managing cheating and plagiarism strategically.

3. The model

We consider two risk neutral agents, each of whom is required to perform a task individually. Performance for each agent is affected by the amount of information the agent possesses and is given by

$$x_i = e_P + e_i + m \cdot \max(c_i \cdot (e_j - e_i), 0) + u_i, \tag{1}$$

where e_P is information agents acquire free of cost; e_i is agent i 's information above and beyond e_P acquired by costly effort; and u_i is an idiosyncratic shock such that $Eu_i = 0$

and $Var(u_i)$ is finite. Performance of agent i can also be enhanced by a risky information leak from agent j , a form of cheating. The latter is captured in (1) by the factor $m \cdot \max(c_i \cdot (e_j - e_i), 0)$, where $c_i \in [0, 1]$ is a choice variable for agent i that stands for the proportion of maximum possible differential of information from effort $(e_j - e_i)$ that i will request from j , as long as this differential is positive. The parameter $m > 0$ captures the exogenous rate at which the information requested will be transformed into performance. We assume that information is purely quantitative. That is, if $e_i \geq e_j$, i already knows more than j and any information transmission from j to i will have no effect on the quantity of i 's information.

Agents are heterogeneous in their ability a_i which is common knowledge and negatively affects the cost of effort for agent i , given by

$$C_i(e_i) = \frac{e_i^2}{2a_i}. \quad (2)$$

Agents make their choices in two stages:

- At *stage 1* agents select their effort levels, e_i and e_j .
- At *stage 2* the agent who has exerted the lower effort (i or j) chooses c , the proportion of the effort differential (cheating rate) she will request.

Agents are also heterogeneous concerning their willingness to facilitate cheating. Specifically, an agent i is of type $\theta_i = 0$ when she declines any request to transmit information to another agent. Conversely, i is of type $\theta_i = 1$ when she fully supplies the amount of information requested by another agent to the means of her capacity, e_i . In our baseline model θ_i is an exogenous parameter and is private information to agent i . Agent j is unaware of the actual value of θ_i but knows that $pr(\theta_i = 1) \equiv q$. Generally, the distribution of types will depend on the environment the agents act within. In some environments agents are socially expected to help those who are weaker and doing so yields a benefit for them. In such environments, q is expected to be higher because this benefit will counterweigh the risk of getting caught cheating for most agents. In other environments, enabling cheating is perceived as an unfair and unethical act, and thus for the majority of agents the benefit from helping will not offset the risk, dropping q close to 0. In section 8 we present an extension where it is allowed to the supplier to select her type and the amount of information she will transmit.

Given (1) and (2), the agent with the lowest ability will de facto exert the lowest effort at stage 1 and will become the demander of cheating at stage 2. For the rest of the analysis, we will be indexing this agent by L . Accordingly, the agent with the highest ability will exert the highest effort at stage 1, and therefore will be the potential supplier of cheating at stage 2. We will use H to index the agent of higher ability for whom $\theta = 1$, and H' to

distinguish the agent of higher ability for whom $\theta = 0$. Since the ability of agents is common knowledge, both agents can be expected to figure out who will be the demander and who will be the potential supplier of information by stage 2.⁵

From the above, it follows that exchange of information will take place depending exclusively on the type of the supplier. An agent's θ becomes irrelevant as she takes the role of the demander at stage 2 because θ pertains only to how a supplier handles requests for cheating.⁶ Interestingly, θ creates an informational asymmetry between the agents differentiating them further than they already are from having potentially different abilities. Entering stage 2, H knows whether she will grant or deny the request for cheating that she will surely receive. On the contrary, L is unaware whether she deals with an agent who will accommodate cheating or not.

Upon completion of the task, the principal compensates the agents according to their performance based on a pre-announced compensation scheme. Compensation for the i^{th} agent is given by

$$w_i = \begin{cases} x_i - b \cdot x_j, & \text{if cheating is not detected,} \\ 0, & \text{if cheating is detected} \end{cases}, \quad (3)$$

where b is the coefficient for the *power of relative incentives* and can reasonably take values in $[0, 1]$. This compensation scheme allows us to consider how cheating works in both absolute and relative grading systems depending whether $b = 0$ or $0 < b \leq 1$. According to (3), if cheating is detected, both agents are punished by being deprived of their entire compensation including the portion that had resulted from legitimate effort. The ex-post probability of detecting cheating is given by

$$p(c) = \frac{c \cdot (e_H - e_L)}{\kappa}, \quad (4)$$

where $\kappa > 0$ is a random variable, inversely related to the ability of the proctor to identify cheating. The actual value of κ is not realized by the agents until the completion of stage 2. However, both agents know ex-ante that $E(\kappa) = k$. Notice that at stage 2, L will never choose $c \geq k / (e_H - e_L)$, as she knows $e_H - e_L$, and doing so will simply expose her to additional risk of being caught without any benefit. Another property of (4) is that it yields by default that $p(0) = 0$. That is, there is no possibility of falsely accusing agents for cheating. Also, $\partial p(c) / \partial (c \cdot (e_H - e_L)) > 0$ implies that it becomes more likely for the agents to be caught as the total amount of information transmitted increases, since the exposure of

⁵We do not consider the case where two equal ability agents are matched as there would be no room for information transmission in such case. Moreover, if a_i and a_j are assumed to be drawn from continuous distributions, the probability of such an event is 0.

⁶Allowing for the demander's θ to affect her engagement in cheating would not alter our results qualitatively. It would simply proportionally mitigate the supplier's exposure to the prospect of cheating.

illegitimate activity to the proctor's attention is longer or more intense. On the other hand, $\partial p(c)/\partial k < 0$ indicates that the lower the ability of the proctor to detect cheating, the lower will be the risk of getting caught.⁷

Agent i 's payoff is given by

$$U_i = w_i - C_i(e_i). \quad (5)$$

Agents are exogenously assumed to have accepted the terms of the agreement, and hence there is no participation issue.

The equilibrium levels of effort e_L^* , e_H^* and the equilibrium cheating c will be affected by the capacity constraint of available information

$$0 \leq c \leq 1. \quad (6)$$

This constraint has 3 possible states: (i) The *unconstrained* state where the (6) is non-binding. We will be indexing this case with the subscript 'A'. (ii) The *cheating at capacity* state where (6) is binding from above. That is, if the optimization problem of L was solved with no constraint, it would yield $c > 1$. We distinguish this case using the subscript 'B'. (iii) The *opt-out* state where (6) is binding from below in the sense that the unconstrained problem would yield $c < 0$. This case will be identified using the subscript 'O'. In what follows, we will proceed by solving for the optimal choices of the agents under the initial assumption that (6) is non-binding (state A). If those optimal choices violate the constraint from above, we will override them with the optimal choices calculated under state B. If they violate (6) from below, we will override them with the optimal choices calculated under state O.

4. Optimal efforts and cheating in different states

We will calculate c and the optimal effort for each potential agent H' , H , L in states A, B and O.⁸

4.A. Unconstrained optimal efforts and cheating (state A)

The unconstrained problem for agent H' (with $a_i = a_H$ and $\theta = 0$)

Agent H' will turn down any request for cheating by the lower ability agent independently

⁷Note that the proctor cannot deduct cheating and punish the agents after observing an unusually high level of performance by L because of the existence of u_L , which is unobservable to the proctor.

⁸In all following derivations all payoff functions are concave with respect to the choice variables. Therefore, the first order conditions (FOC) are necessary and sufficient for the optimization.

of the level of c . The expected payoff for H' is

$$EU_{H'}(e_{H'}) = e_{H'} - be_L + (1 - b)e_P - \frac{e_{H'}^2}{2a_H}. \quad (7)$$

H' solves

$$\max_{e_{H'}} EU_{H'}(e_{H'}) \quad (8)$$

$$\text{s.t. } e_{H'} \geq 0. \quad (9)$$

The first order condition implies

$$e_{H'}^* = a_H, \quad (10)$$

which always satisfies the constraint (9) as $a_H > 0$. According to (10), the optimal effort for H' is directly proportional to her ability. This is a standard result in the agency literature. Moreover, her choice of effort is independent of e_P , since the costless information endowment of an agent does not affect the marginal cost of acquiring information.

The unconstrained problem for agent L (with $a_i = a_L$)

Agent L will choose e_L at stage 1 and c at stage 2. The expected payoff for L at stage 2 is

$$\begin{aligned} EU_{L,A}(e_L, c) &= (1 - q)(e_L + (1 - b)e_P - be_{H'}^*) + \dots \\ &\dots + q \left(1 - \frac{c(e_H - e_L)}{k} \right) (e_L + (1 - b)e_P + mc(e_H - e_L) - be_H) - \frac{e_L^2}{2a_L}. \end{aligned} \quad (11)$$

The first term in (11) is where L accounts for the probability that has to deal with an H' -match at stage 2, while the second term accounts for an H -match. At stage 2 agent L solves

$$\max_c EU_{L,A}(e_L, c) \quad (12)$$

$$\text{s.t. } 0 \leq c \leq 1. \quad (13)$$

The first order condition implies that

$$c = \frac{be_H - e_L + mk - (1 - b)e_P}{2m(e_H - e_L)}. \quad (14)$$

Equation (14) has several interesting implications. As k has a negative impact on the risk of getting caught, it positively affects the demand for cheating, c . Similarly, m has a positive impact on the value of information transmitted, and thus it also affects c positively. Conversely, the demand for cheating decreases in e_L because cheating and effort are substitutes concerning L 's performance. More interestingly, when the grading system is relative ($b > 0$)

the effect of e_H on c is non-monotonic. As e_H increases, on the one hand, there is more information available to be transmitted at a risk, so c can be lowered. On the other hand, the relative component in evaluation increases the need for information to close the gap $x_L - bx_H$ which decreases the demander's payoff.

At stage 1 L chooses her effort. Substituting (14) in (11) we derive the expected payoff for L at stage 1

$$EU_{L,A}(e_L) = (1 - q)(e_L + (1 - b)e_P - be_{H'}^*) + \dots$$

$$\dots + \frac{q}{4mk}(e_L + (1 - b)e_P - be_H + mk)^2 - \frac{e_L^2}{2a_L}. \quad (15)$$

Agent L solves

$$\max_{e_L} EU_{L,A}(e_L) \quad (16)$$

$$\text{s.t. } e_L \geq 0. \quad (17)$$

The first order condition satisfies

$$e_L(e_H) = a_L \frac{mk(2 - q) + q(1 - b)e_P - bq e_H}{2mk - a_L q}. \quad (18)$$

Condition (18) is the incentive compatibility constraint for L as it ensures that she maximizes her expected payoff given the effort choice of H . Moreover, there is no participation issue, and thus the individual rationality constraint is satisfied exogenously. Observe that, when a relative scheme is in use, the direct effect of e_H on e_L is negative. This is because as b increases, the marginal benefit from e_L decreases, causing L to substitute effort with cheating. Given that c is already determined by (14), the only way to increase the quantity of cheating $mc(e_H - e_L)$ is by lowering e_L . Since choices of e_L and e_H are made simultaneously, we must derive the incentive compatibility for H in order to tell if (18) violates (13).

The unconstrained problem for agent H (with $a_i = a_H$ and $\theta = 1$)

Agent H will choose e_H at stage 1 taking into account L 's choice of e_L at the same stage and foreseeing L 's choice of c at stage 2, which will expose her to risk of getting caught because of her type. The expected payoff for H is

$$EU_{H,A}(e_H) = \left(1 - \frac{c(e_H - e_L)}{k}\right) (e_H + (1 - b)e_P - b(e_L + mc(e_H - e_L))) - \frac{e_H^2}{2a_H}. \quad (19)$$

Foreseeing c from stage 2, H substitutes (14) into (19) and her expected payoff becomes

$$EU_{H,A}(e_H) = \frac{-be_H + (1-b)e_P + mk + e_L}{4mk} \dots$$

$$\dots \cdot ((2-b^2)e_H + (2+b)(1-b)e_P - bmk - be_L) - \frac{e_H^2}{2a_H}. \quad (20)$$

H solves

$$\max_{e_H} EU_{H,A}(e_H) \quad (21)$$

$$\text{s.t. } e_H \geq 0. \quad (22)$$

The first order condition implies that

$$e_H(e_L) = a_H \frac{(1-b)(1-b-b^2)e_P + mk + e_L}{2mk + a_H b(2-b^2)}. \quad (23)$$

Similarly to (18), (23) describes the incentives for the higher ability agent. H responds positively to e_L because the higher the e_L , the less cheating L will demand, decreasing the exposure to risk for both agents.

Unconstrained choices of effort and cheating

Solving the system of (18) and (23) we obtain

$$e_{H,A}^* = a_H \frac{(1-b)(2mk - b(b+1)(2mk - a_L q))e_P + 2mk(mk + a_L(1-q))}{(2mk + a_H b(2-b^2))(2mk - a_L q) + a_H a_L b q}. \quad (24)$$

Substituting (24) into (18) we can derive the optimal effort for the low ability agent

$$e_{L,A}^* = a_L \frac{mk(2-q) + q(1-b)e_P - bq e_{H,A}^*}{2mk - a_L q} \quad (25)$$

and by substituting (24) and (25) into (14) we obtain the equilibrium unconstrained rate of cheating

$$c_A^* = \frac{be_{H,A}^* - e_{L,A}^* + mk - (1-b)e_P}{2m(e_{H,A}^* - e_{L,A}^*)}. \quad (26)$$

Agent H' does not interact with her counterpart in any stage, thus her choice of effort is always given by (10).

4.B. Cheating at capacity (state B)

The optimal amount of cheating defined in (26) may violate the constraint (13) upwards, that is $c_A^* > 1$. Then, the set of choice variables in (24), (25) and (26) are not feasible.

In such case, agents L and H will take into account that c has to be set to its maximum possible value of 1 and will re-optimize as follows. The choice of $e_{H'}$ by agent H' will not be affected by (13) and remains as in (10).

The constrained problem for agent L (with $a_i = a_L$)

When (26) violates (13) from upwards, at stage 2 agent L sets

$$c_B^* = 1 \quad (27)$$

and her expected payoff becomes

$$\begin{aligned} EU_{L,B}(e_L) &= (1 - q)(e_L + (1 - b)e_P - be_{H'}) + \dots \\ &\dots + \frac{q}{k}(k - e_H + e_L)((1 - m)e_L + (1 - b)e_P + (m - b)e_H) - \frac{e_L^2}{2a_L}. \end{aligned} \quad (28)$$

At stage 1 agent L solves

$$\max_{e_L} EU_{L,B}(e_L) \quad (29)$$

$$\text{s.t. } e_L \geq 0 \quad (30)$$

for which the first order condition implies

$$e_L(e_H) = a_L \frac{k(1 - qm) + q(1 - b)e_P + q(2m - 1 - b)e_H}{k - 2a_Lq(1 - m)}. \quad (31)$$

Interestingly, when the power of relative incentives is high, that is $b > 2m - 1$, the negative direct effect of e_H on e_L observed in (18) from the unconstrained problem persists. In the opposite case where the power of relative incentives is sufficiently low and $b < 2m - 1$, the direct effect will be reversed. This is because c is already at its maximum point and the only way for L to close the gap $x_L - bx_H$, which decreases her payoff, is by exerting higher effort.

The constrained problem for agent H (with $a_i = a_H$ and $\theta = 1$)

Agent H will be able to foresee that (13) is violated upwards and will modify her expected payoff according to (27);

$$EU_{H,B}(e_H) = \frac{1}{k}(k - e_H + e_L)((1 - bm)e_H + (1 - b)e_P + b(m - 1)e_L) - \frac{e_H^2}{2a_H}. \quad (32)$$

Agent H solves

$$\max_{e_H} EU_{H,B}(e_H) \quad (33)$$

$$\text{s.t. } e_H \geq 0. \quad (34)$$

The first order condition implies

$$e_H(e_L) = a_H \frac{(1 - bm)k - (1 - b)e_P + (1 - b(2m - 1))e_L}{k + 2a_H(1 - bm)}. \quad (35)$$

When there is excess demand for cheating -similarly to the unconstrained case- e_H responds to e_L weakly positively because $1 - b(2m - 1) \geq 0 \forall b \in [0, 1]$ and $\forall m \in [0, 1]$.

Constrained choices of effort

Solving the system of (31) and (35) we obtain

$$e_{L,B}^* = a_L \frac{F_1 + F_2}{F_3}, \quad (36)$$

where

$$\begin{aligned} F_1 &= k(k(1 - qm) + a_H(1 - bm)(2 - q(1 + b))), \\ F_2 &= q(1 - b)(k + a_H(3 - 2m(1 + b) + b))e_P, \end{aligned}$$

and

$$F_3 = (k - 2a_Lq(1 - m))(k + 2a_H(1 - bm)) - a_Ha_Lq(2m - 1 - b)(1 - b(2m - 1)).$$

Substituting (36) into (35) we can derive the optimal effort for the high ability agent

$$e_{H,B}^* = a_H \frac{k(1 - bm) - (1 - b)e_P + (1 - b(2m - 1))e_{L,2}^*}{k + 2a_H(1 - bm)}. \quad (37)$$

Again, choice of effort for H' is still given by (10).

4.C. Opt-out from cheating (state O)

If the optimal amount of cheating defined in (26) yields $c_A^* < 0$, agent L will voluntarily select to request no information from H . Again, $e_{H'}$ will remain as in (10).

The opt-out problem for agent L (with $a_i = a_L$)

At stage 2 L will set

$$c_O^* = 0. \quad (38)$$

At stage 1, expected payoff for L becomes

$$EU_{L,O}(e_L) = e_L + (1 - b)e_P - b(1 - q)e_{H'} - bq e_H - \frac{e_L^2}{2a_L} \quad (39)$$

Agent L solves

$$\max_{e_L} EU_{L,O}(e_L) \quad (40)$$

$$\text{s.t. } e_L \geq 0. \quad (41)$$

The first order condition implies that

$$e_{L,O}^* = a_L. \quad (42)$$

The opt-out problem for agent H (with $a_i = a_H$ and $\theta = 1$)

Agent H will be able to foresee that L will request zero cheating and will modify her expected payoff as

$$EU_{H,O}(e_H) = e_H - be_L + (1 - b)e_P - \frac{e_H^2}{2a_H}. \quad (43)$$

Then, H solves

$$\max_{e_H} EU_{H,O}(e_H) \quad (44)$$

$$\text{s.t. } e_H \geq 0. \quad (45)$$

The first order condition yields

$$e_{H,O}^* = a_H. \quad (46)$$

Condition (46) implies that the choice by H will be equivalent to that of H' from (10).

5. Equilibrium

From the results of the previous sections, the choices of the agents will be $(e_{H,A}^*, e_{L,A}^*, c_A^*)$ or $(e_{H,B}^*, e_{L,B}^*, c_B^*)$ or $(e_{H,O}^*, e_{L,O}^*, c_O^*)$, depending on the state of the constraint (13). However, (13) can be affected by the agents, directly by L 's choice of c and indirectly by the level of e_H because, in conjunction with e_L , e_H sets the maximum capacity for cheating. Therefore, agents may want to adjust effort strategically in order to install the particular state of the constraint that serves their personal interests. We will now cover all possible outcomes.

Lemma 1: For $a_i > 0$, $m > 0$, $b \in [0, 1]$, $k > 0$, $e_P > 0$ and $q \in [0, 1]$, it is $EU_{H,O}(e_{H,O}^*) > EU_{H,A}(e_{H,A}^*)$ and $EU_{H,O}(e_{H,O}^*) > EU_{H,B}(e_{H,B}^*)$.

Proof. Relationships (43), (19) and (32) demonstrate that the effect of $c > 0$ decreases the expected payoff for H in comparison to $EU_{H,O}$. ■

This Lemma simply states that agent H always prefers state O to any other state.

Lemma 2: For $a_i > 0$, $m > 0$, $b \in [0, 1]$, $k > 0$, $e_P > 0$ and $q \in [0, 1]$, it is $EU_{L,A}(e_{L,A}^*) >$

$EU_{L,O}(e_{L,O}^*)$ and $EU_{L,A}(e_{L,A}^*) > EU_{L,B}(e_{L,B}^*)$.

Proof. In relationships (28) and (11) L prefers setting $c^* > 0$ to $c = 0$, while the latter is still feasible. ■

Lemma 2 states that the most preferable state for L is state A.

Lemmas 1 and 2 will be useful in proving the following propositions for all values of the parameters $a_i > 0$, $m > 0$, $b \in [0, 1]$, $k > 0$, $e_P > 0$ and $q \in [0, 1]$ that yield non-negative $e_{H,A}^*$, $e_{H,B}^*$, $e_{L,A}^*$, and $e_{L,B}^*$.

Proposition 1: *If $EU_{L,O}(e_{L,O}^*) > EU_{L,B}(e_{L,B}^*)$ when $c_A^* > 1$, then the game will end up at state O and the equilibrium will be $(e_{H,O}^*, e_{L,O}^*, c_O^*)$.*

Proof. Given $c_A^* > 1$ state A is not feasible for L . If therefore state O is preferable to state B for L , L will choose $c^* = 0$, allowing H to reach her preferable state O as shown in Lemma 1. ■

The lower ability agent will prefer to voluntarily give up cheating entirely if her constrained payoff is lower than her payoff without cheating. This can happen when L cannot afford to take the risk of getting caught for the quantity of information that H is going to make available. As it can be seen in Figure 1 (panels (a) and (b)), this will be the case when a_H is slightly higher than a_L (that is, near the origin in panels (a) and (a)) of Figure 1. In response to L 's choice, H will be relieved from the risk associated with enabling cheating and will set her effort all the way up to $e_{H,O}^*$.

Proposition 2: *If $EU_{L,B}(e_{L,B}^*) > EU_{L,O}(e_{L,O}^*)$ and $c_A^* \leq 0$, then the game will end up at state O and the equilibrium will be $(e_{H,O}^*, e_{L,O}^*, c_O^*)$.*

Proof. According to Lemma 2, state A is preferable to B for L , and thus she will choose $c = 0$, which will enable H to exert $e_{H,O}^*$. The equilibrium will be equivalent to that of state O. ■

This outcome will be the case for intermediate differentials $a_H - a_L$, such as the rest of the left shaded area in panels (a) and (b) of Figure 1.

Proposition 3: *If $EU_{H,B}(e_{H,B}^*) > EU_{H,A}(e_{H,A}^*)$ and $c_A^* > 0$, then the equilibrium outcome will be $(e_{H,B}^*, e_{L,B}^*, c_B^*)$.*

Proof. If L sets $c > 0$ and H prefers state B over state A, then H will choose $e_{H,B}^*$ forcing L to choose $e_{L,B}^*$ and c_B^* . ■

In this case, the higher ability agent will act strategically and prefer to lower her effort to a sub-optimal level, as far as the unconstrained problem is concerned, in order to limit her capacity and cap the information flow towards the lower ability agent. The moderation of her effort will make the higher ability agent incur a direct cost in her expected performance

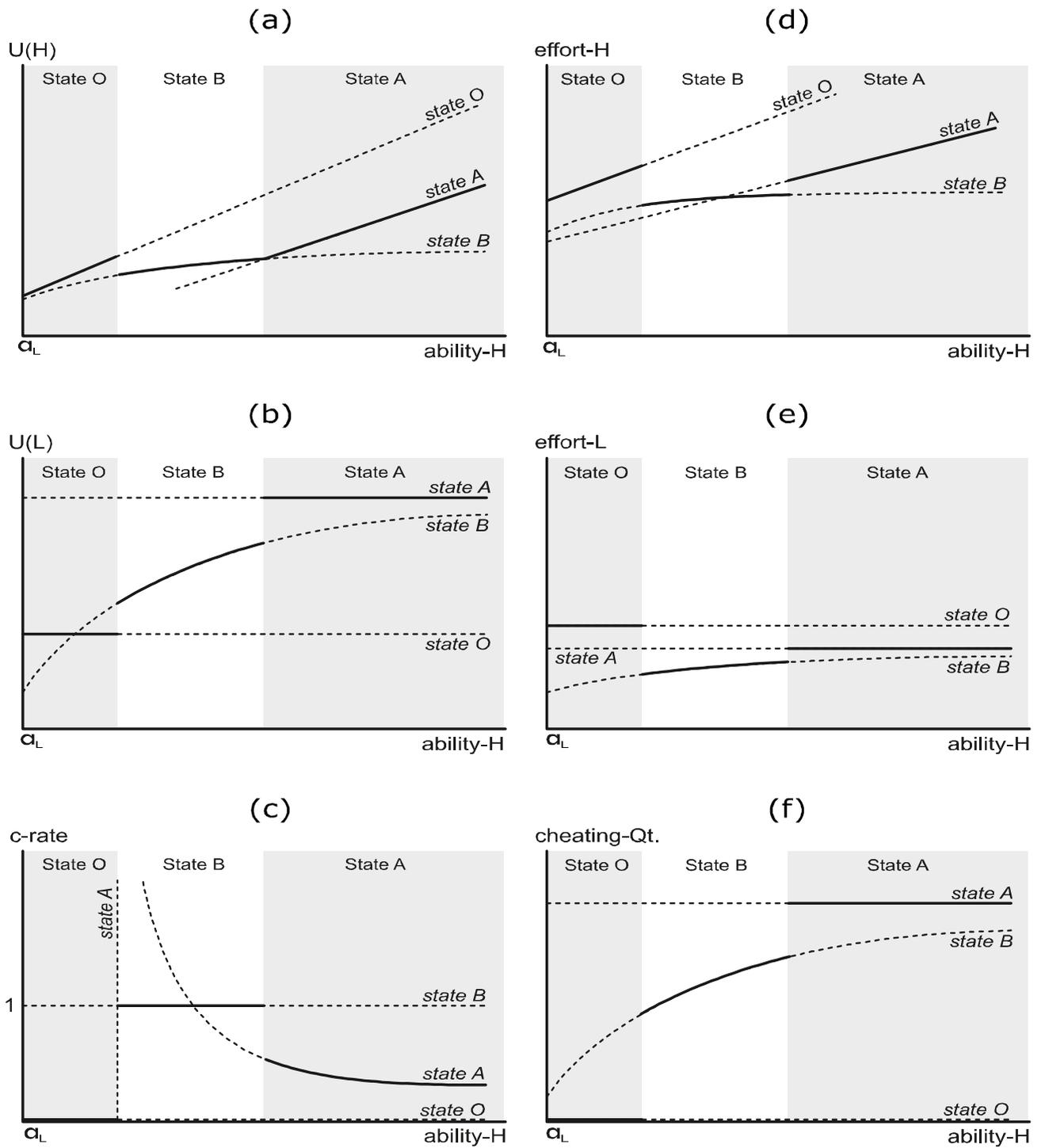


Figure 1: Changes in payoffs, efforts and cheating as a_H increases.

but this will be offset by the benefit of controlling the risk of getting caught cheating. L will foresee that H will exert $e_{H,B}^*$ (even though $e_{H,A}^*$ is feasible) and L 's best response will be to exert $e_{L,B}^*$, pushing an otherwise unconstrained problem to its constrained state. This will also be the case for intermediate differentials $a_H - a_L$ but higher than in the previous case, as shown in the unshaded area of panels (a) and (b) of Figure 1.

Proposition 4: *If $EU_{H,A}(e_{H,A}^*) > EU_{H,B}(e_{H,B}^*)$ when $c_A^* \in [0, 1]$, the equilibrium outcome will be $(e_{H,A}^*, e_{L,A}^*, c_A^*)$.*

Proof. If $EU_{H,A}(e_{H,A}^*) > EU_{H,B}(e_{H,B}^*)$, H will choose $e_{H,A}^*$ allowing L to set $e_{L,A}^*$ and c_A^* . ■ The high ability agent cannot afford to constrain the quantity of information transmission to the low ability agent by choosing $e_{H,B}^*$ instead of $e_{H,A}^*$. That is, H is required to decrease effort significantly in order to cap cheating installing state B. Thus, it is worth for H to take the risk rather than losing a high amount of expected performance from the drop in effort. This will be the case when the ability of H is disproportionately higher than that of L , as in the right shaded side of panels (a) and (b) of Figure 1.

Proposition 5: *If $EU_{L,O}(e_{L,O}^*) > EU_{L,B}(e_{L,B}^*)$ and $EU_{H,B}(e_{H,B}^*) > EU_{H,A}(e_{H,A}^*)$ when $c_A^* \in [0, 1]$, then no Nash Equilibrium in pure strategies exists in the game between the two agents.*

Proof. For H , $e_{H,A}^*$ is a strictly dominated strategy. If H tries to constrain L by exerting $e_{H,B}^*$, L will prefer to respond with $e_{L,O}^*$, and then H will also want to deviate to $e_{H,O}^*$. If H exerts $e_{H,O}^*$, L will want to respond with $e_{L,A}^*$, which will make H to want deviate to $e_{H,B}^*$. ■

This is a peculiar case, where H will mix between $e_{H,B}^*$ and $e_{H,O}^*$, while L will mix between $e_{L,A}^*$ and $e_{L,B}^*$. This case will occur only for a few particular combinations of the parameters and for a very limited range.

The propositions above lead us to the following theorem.

Theorem 1: *For $a_i > 0$, $m > 0$, $b \in [0, 1]$, $k > 0$, $e_P > 0$ and $q \in [0, 1]$ that yield non-negative $e_{H,A}^*$, $e_{H,B}^*$, $e_{L,A}^*$, and $e_{L,B}^*$:*

- *The equilibrium is $(e_{H,O}^*, e_{L,O}^*, c_O^*)$ if the conditions of proposition 1 or proposition 2 are satisfied.*
- *The equilibrium is $(e_{H,A}^*, e_{L,A}^*, c_A^*)$ if the conditions of proposition 4 are satisfied.*
- *The equilibrium is $(e_{H,B}^*, e_{L,B}^*, c_B^*)$ if the conditions of proposition 3 are satisfied.*
- *There is no pure strategy equilibrium if the conditions of proposition 5 are satisfied.*

Proof. Proof follows from individual propositions 1-5. ■

6. Comparative statics

It would be useful to discuss the comparative statics for e_H^* , e_L^* , and c^* in all three possible states of the problem. This will provide us with the necessary intuition background in order to understand the equilibrium outcome. We will split the analysis into two parts; first, we will consider the simpler case of absolute compensation when $b = 0$; then, we will advance to the case where performance is evaluated in a relative manner, where $b > 0$, and we will compare it to the absolute benchmark. In what follows, we discuss the behavior of the equilibrium (e_H^*, e_L^*, c^*) according to constraint (13), as e_H^* is defined in (24), (37) or (46); e_L^* is defined in (25), (36) or (42) and c^* is defined in (26) (27) or (38).

6.A. Comparative statics for absolute compensation

Ability

The optimal effort for the higher ability agent within any state of the problem will always be increasing with her ability as increased ability makes effort less costly. The same is true for the lower ability agent. Moreover, agent L will positively respond to changes of a_H when demand for cheating is excessive in state B. According to (28) and as it can be seen from the unshaded areas in panels (c), (d), (e) and (f) of Figure 1, when $c = 1$, L can only adjust the information requested, $c \cdot (e_H - e_L)$, through e_L . As e_H increases, $c \cdot (e_H - e_L)$ increases faster causing L to raise her effort. In other words, a higher e_H drives L to trade cost of effort for a lower exposure to risk. Once L reaches the unconstrained optimal level of information she wishes to copy, she will want to gradually decrease c in order to keep $c \cdot (e_H - e_L)$ at that level. Therefore, in the unconstrained state A she will not adjust her effort with the ability of H as in the right shaded area of panels in Figure 1.

Transformation rate of cheating

The transformation rate, m , affects e_H in a negative manner when cheating is involved. As m becomes higher, the marginal value of information transmitted increases for L , making her more willing to tolerate the risk to acquire it. As a result, L will have the tendency to increase cheating (both the rate, c , and the quantity $c \cdot (e_H - e_L)$) and to decrease her effort. This over-exposes H to risk, causing her to moderate her own effort in an attempt to restrict the quantity of information transmitted, and thus counter-balance the risk. A sufficient increase in m may cause the outcome to cross over from the unconstrained state A to the constrained state B.

Probability of $\theta = 1$

Probability q negatively affects the optimal effort by either agent. When the lower ability agent is more certain that she will encounter a match who will facilitate cheating at stage

2, she becomes more willing to risk cheating in order to avoid costly effort at stage 1. This, however, puts H more in jeopardy to get caught and lose her entire compensation. Her only way to limit this risk is by cutting back on effort.

Information endowment e_P

Agent L responds to an exogenous handout of free effort by increasing her own, independently of the state of the constraint. This is because the higher the e_P the more is at stake to be lost in case she is caught cheating. Agent H , however, will behave differently depending on the state of the constraint. For sufficiently low e_P the game is likely to end up in state B. There, H will want to decrease costly e_H as she receives more costless e_P because doing otherwise may bring her to state A, where she faces a higher risk. If e_P rises sufficiently, the game will indeed enter state A because cheating activity subsides as L increases effort and decreases cheating. At this point, H can increase her effort without affecting her risk. There exists a sufficiently high value of e_P , above which the game will pass to state O, cheating will be completely eliminated and both agents will prefer to acquire payoff with honest means since the risk of punishment will no longer be tolerable.

6.B. The effect of relative compensation

We consider now the case where the compensation of each agent depends on the performance of her rival. This effectively means that the compensation system penalizes the higher effort agent for providing information to the lower effort agent. Expected compensation for agent H in case she is not caught cheating is

$$Ew_H = (e_P + e_H) - b(e_L + e_P) - cbm(e_H - e_L). \quad (47)$$

The first part of (47) is the compensation H would receive had there been no relativity in the scheme and no cheating. The second term is the reduction in compensation that arises from the relativity of the scheme and the third factor is the penalty H incurs for releasing information to L . As it is apparent from (47), the penalty for cheating is dependent upon the implementation of a relative compensation scheme and it is positively affected by b . Therefore, as b increases, cheating subjects H to a decrease in compensation, which should affect her strategic choice of e_H .

On the other hand, the situation for L is quite different. Expected compensation for L in case she is not caught cheating is

$$Ew_L = (e_P + e_L) - b(e_H + e_P) + cm(e_H - e_L). \quad (48)$$

This means that, while the penalty from relativity is higher for L than for H , L also receives

a benefit from cheating which is not affected by b . Agent L 's attitude towards cheating will, therefore, be affected by her attempt to counterbalance two effects: first, her direct loss from the penalty by the relative scheme $b(e_H + e_P) > 0$; second, the change in cheating capacity that will result from the response of her counterpart's effort to the higher-power incentive scheme.

As it can be seen in Figure 2 (panel (c) - left shaded area), when the problem is in its unconstrained state A, b causes an increasing tendency to c . This is because as the penalty from relative incentives $b(e_H + e_P)$ increases, the effort by H decreases (panel (a) - left shaded area) and L responds by increasing the rate of cheating in order to cut down her losses. Moreover, in the usual case when $m < 1$ or $c < 1$, the optimal level of effort for L will decrease and increasing cheating will be her only response to the relative incentives. This means that when the ability differential is sufficiently high to bring the game to its unconstrained state A, the introduction of a relative component in the method of compensation will, firstly, heavily penalize and discourage the high ability agent and, secondly, intensify the cheating problem instead of alleviating it.

If b keeps increasing further, the reduction in e_H will lead the problem to cross over to its constrained state B. As it can be seen in the unshaded areas of Figure 2, L will keep dropping e_L in order to substitute it with more cheating as the constraint has now become binding. H will also further reduce e_H to limit the quantity of cheating that will be transacted, and thus restrict her exposure to risk. At the constrained state B, relative compensation will start alleviating the cheating problem but it will also more aggressively discourage agents to exert effort. In fact, the higher ability agent is the one who will be overwhelmed with penalties from b , to the point that she performs as if she were a low ability agent.

If the abilities of the agents are not too far apart and b becomes sufficiently high, the game may pass to state O and as it can be seen towards the right shaded area of panel (d) in Figure 2, cheating will be completely eliminated. However, the remedy is not coming free of side effects. Payoffs for both agents will be drastically reduced from the severe penalty component of compensation in comparison to an absolute scheme. For the cheating agent, payoff is mainly reduced because she forgoes potential compensation from cheating. This is not a necessarily negative outcome since eliminating cheating was the point of the introduction of a relative system to begin with. However, the higher ability agent is also unfairly penalized -relative to absolute compensation- even though cheating does not finally occur.

7. Optimal matching

An interesting dilemma from a policy perspective is how agents of various abilities should be grouped together in order to alleviate cheating or create an environment that encourages

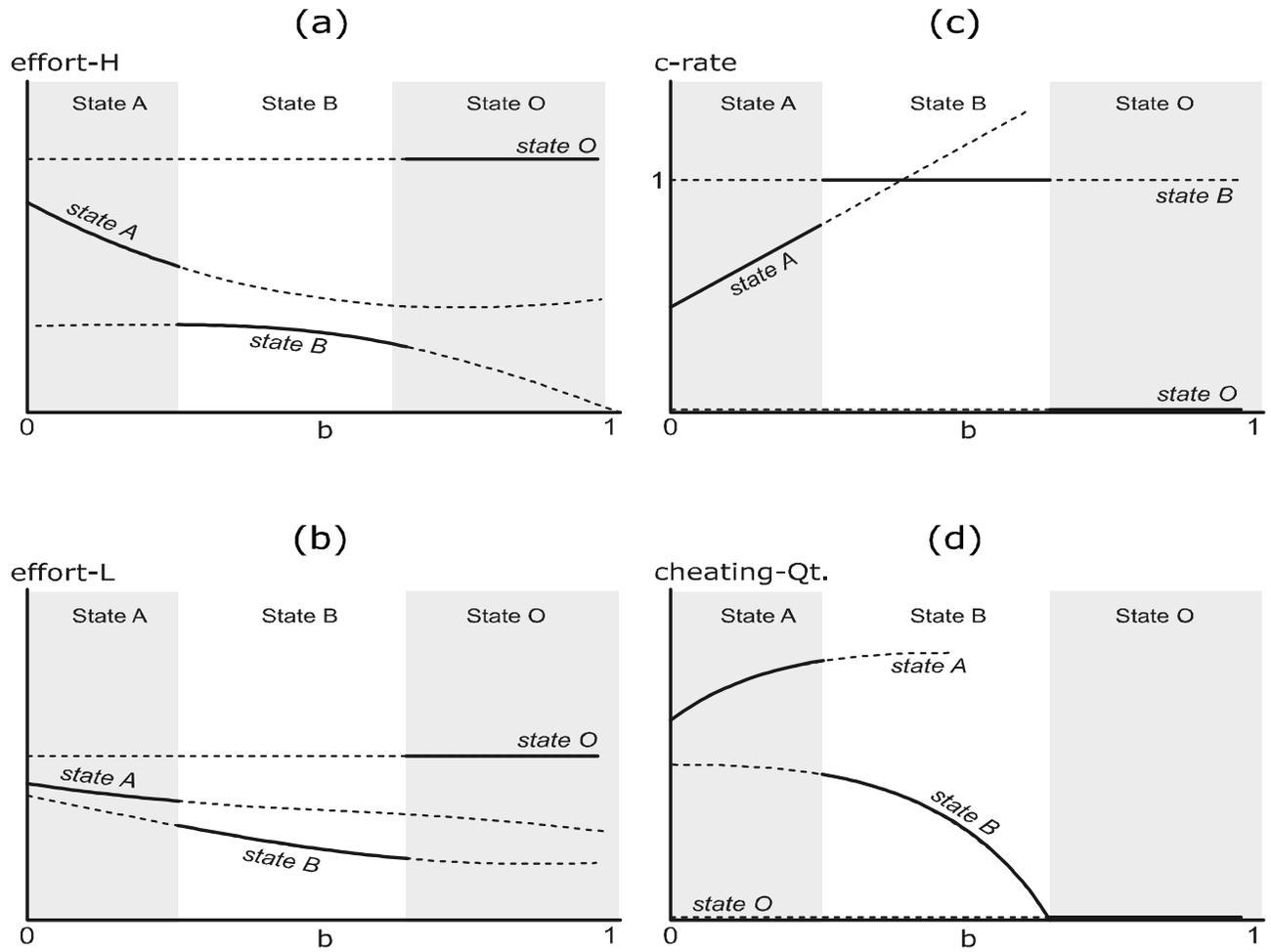


Figure 2: The effect of an increase in b on efforts and cheating.

performance. In education, for instance, some institutions prefer mixing students of different abilities, while others separate higher from lower ability students in honor and regular classes. The comparison between mixing and separating agents of different abilities cannot be definitively addressed without considering synergy effects in the production function or peer effects in the utility functions of agents. Nevertheless, our model is useful in isolating the impact of the “cheating effect” on the optimal choices of agents when they are either mixed or separated.

Figure 1 (panel (d)) shows that as the ability of H increases, making the two agents more heterogeneous, the higher ability agent increases her effort (see (37) and (24)), yet she increasingly deviates from the effort she would have exerted if cheating was not an issue (see (46)). This is true regardless if the state of the constraint is A or B. In other words, a cheating culture hurts more a supplier of higher ability than a supplier of lower ability. This is because, the higher the ability of the supplier, the more information she will be potentially asked to transmit, adding more risk to her marginal cost of effort, and thus resulting in under-performance. On the other hand, the low ability agent may increase her effort (see (36)) as the two agents become more heterogeneous when the problem is at state B. If the agents become sufficiently heterogeneous, the problem will pass over to state A and L will adjust her effort (see (25)) to a level independent of a_H , higher than that in stage B but still lower than the effort in state O (see (42)).

If we consider $e_L + e_H$, the level of ‘total output’ for the group of agents, we can conclude that heterogeneity of abilities increases total output but causes it to increasingly deviate from the no-cheating total output (that is, the sum of efforts as if the agents were at state O from (46) and (42)). This happens mainly because of the disproportional decrease in the effort of the supplier. We can therefore conclude that in environments that the cheating problem is severe, agents must be separated in groups of similar abilities. In fact, when abilities are not sufficiently different, the problem will pass to state O because the risk of sharing information will overshadow its benefit. As a result, separation of agents is the only cost-free way to deter cheating.

It is also worth noticing in Figure 1 (panel (b)) that cheating -regardless of state- clearly improves the payoff of the lower ability agent. This is because the prospect of cheating allows L to beneficially trade expensive cost of effort for relatively cheaper risk of being caught. Conversely, cheating negatively affects the payoff of the higher ability agent irrespective of whether it occurs at state A or at state B (see panel (a)). However, it is necessary to point out that the payoff of the higher ability agent is net of the social benefit she receives from “helping” her counterpart. We will extend the model in order to capture this exact benefit in the next section.

8. Type-switching

The preceding finding regarding the devastating effect of relative performance evaluation on the higher ability agent's effort raises a natural question. Why does she not limit her cheating supply in order to curb her losses from the relative component of compensation? We will now generalize our model to include the case, where the higher ability agent is allowed to control the amount of information she supplies to the lower ability agent. This extension will effectively shed light on the process behind the determination of θ for the agent who acts as the supplier. That is, how and whether the high ability agent chooses to become type H or type H' .

We maintain all assumptions of our baseline model of section 3. However, the payoff of the high ability agent now becomes

$$U_H = w_H - C_H(e_H) + B(s), \quad (49)$$

where $s \in [0, 1]$ is the rate of cheating from the supply side. That is, the maximum portion of $(e_H - e_L)$ that H supplies to L . The term $B(s)$ is the benefit H receives from the mere fact that she helps L . In general, a positive supply of cheating is an act motivated in a social context and possibly involves some form of altruism.⁹ An agent for whom $B \leq 0$ will always want to set cheating supply to zero as any involvement in cheating would be a strictly dominated strategy for her. This is because, even when $B = 0$ cheating will increase the danger of getting caught and will effectively bring her expected compensation to a worse state. The expected payoff for the high ability agent will, then, become

$$EU_H = \left(1 - \frac{s \cdot (e_H - e_L)}{k}\right) [e_H + e_P - b(e_L + e_P + m \cdot s(e_H - e_L))] - \frac{e_H^2}{2a_H} + B(s). \quad (50)$$

We specify the functional form of the social benefit as

$$B(s) = s^2 \cdot (1 - b) \cdot A, \quad (51)$$

where A is an exogenous benefit for the supplier. In (51) we discount A with $(1 - b)$ because, naturally, the relative component of compensation removes some of the benefit. Under absolute compensation, an agent with $s = 1$ will enjoy the full social benefit. Under relative compensation, however, an agent who grants access to unlimited cheating leaves herself open to her counterpart for exploitation resulting in a decrease of her payoff. In reality, cooperative cheating is a phenomenon driven by peer effects among agents. Based on this

⁹See papers from section 2 and more specifically, Crown and Spiller (1998) and Magnus, Polterovich, Danilov and Savvateev (2002).

logic, the relative scheme provides a socially acceptable excuse for the higher ability agent to limit her cheating supply because her peers can understand that otherwise she actively reduces her own payoff. On the contrary, under an absolute scheme, the higher ability agent incurs only the risk of getting caught helping another agent, which usually is not a socially justifiable excuse to limit cheating supply.¹⁰

Notice that (50) is convex with respect to the supply rate of cheating s . This means that the maximization problem for H results only in the corner solutions $s^* = 0$ or $s^* = 1$, depending on the parameters. Therefore, as a matter of fact, s^* will coincide with the agent's binary parameter of ethicality, θ . In this manner, the generalized version of our model in this section describes how the agent with the higher ability decides among types H or H' .

In stage 1 of the generalized model, agents choose e_H and e_L . In stage 2, agent H chooses s , while agent L chooses d , the rate of cheating she will demand. Then, the amount of cheating that will be actually transacted in the end of the game is

$$c \cdot (e_H - e_L) = \min\{d, s\} \cdot (e_H - e_L). \quad (52)$$

¹¹ The extended model has several cases and sub-cases depending on whether there is excess demand ($d > s$) or not; and whether the constraint $0 \leq d \leq 1$ is binding or not. However, the particular type switching from $\theta = 1$ toward $\theta = 0$ as it was discussed in the previous section, will only occur under a single specific case, namely when d takes its corner value equal to s and thus, $c = d = s$.¹² In this case, cheating is scarce causing L to request as much information as possible from H . Therefore, H virtually controls c by choosing between $s = 0$ or $s = 1$, which effectively corresponds to a choice of type, θ . When $d = s \equiv \theta$, the lower ability agent substitutes θ directly into her expected payoff function, which becomes

$$EU_L = \left(1 - \frac{\theta \cdot (e_H - e_L)}{k}\right) [e_L + e_P + m \cdot \theta (e_H - e_L) - b(e_H + e_P)] - \frac{e_L^2}{2a_L}. \quad (53)$$

Notice that in (53) the probability of $\theta = 1$, denoted by q in the baseline model, is no longer meaningful here because L can deduce whether she deals with an H or H' type.

Solving the problem where H chooses $\theta = 1$ or $\theta = 0$ in the 2nd stage by backward induction, we can distinguish the following cases.

¹⁰Marinakakis (2018) contains an extensive discussion on peer effects in relative evaluation.

¹¹Notice that in this extended version of the model, c does not coincide with d as it was implicitly the case in the baseline version of the model, where d was always considered to be 1.

¹²This is because (i) if $d < s$ the problem is similar to the one of the baseline model in section 3, and (ii) if $d > s$, c will be set by the supply side, which L can foresee at equilibrium and thus will adjust $d = s = c$.

Given that in stage 2 H has chosen $\theta = 1$, in the 1st stage she solves

$$\max_{e_H} EU_H(e_H) \quad (54)$$

$$\text{s.t. } e_H \geq 0. \quad (55)$$

The problem (54) is concave with respect to e_H . The first order condition is sufficient and implies

$$e_H(e_L) = a_H \frac{(1 - bm)k - (1 - b)e_P + (1 - b(2m - 1))e_L}{k + 2a_H(1 - bm)}. \quad (56)$$

At stage 1, again, agent L solves

$$\max_{e_L} EU_L(e_L) \quad (57)$$

$$\text{s.t. } e_L \geq 0 \quad (58)$$

for which the first order condition implies

$$e_L(e_H) = a_L \frac{k(1 - m) + (1 - b)e_P + (2m - 1 - b)e_H}{k - 2a_L(1 - m)}. \quad (59)$$

Solving the system of (59) and (56) we obtain

$$e_L^* = a_L \frac{F_1 + F_2}{F_3}, \quad (60)$$

where

$$F_1 = (k(1 - m) + (1 - bm)e_P)(k + 2a_H(1 - bm)),$$

$$F_2 = a_H(2m - 1 - b)(k(1 - bm) + (1 - b)e_P)$$

and

$$F_3 = (k - 2a_L(1 - m))(k + 2a_H(1 - bm)) - a_H a_L(2m - 1 - b)(1 - b(2m - 1)).$$

Hence,

$$e_H(e_L^*) = a_H \frac{(1 - bm)k - (1 - b)e_P + (1 - b(2m - 1))e_L^*}{k + 2a_H(1 - bm)}. \quad (61)$$

Given that H has chosen $\theta = 0$, we revert to the opt-out choices of effort for both agents (46) and (42).

The higher ability agent will choose $\theta = 1$ over $\theta = 0$ iff

$$\left. \frac{\partial EU_H}{\partial b} \right|_{\theta=1} > \left. \frac{\partial EU_H}{\partial b} \right|_{\theta=0}. \quad (62)$$

That is, the loss in compensation from the relative component is lower under cheating than under no cheating. This leads us to Proposition 6.

Proposition 6: *There exist sets of parameters (a_H, a_L, m, k, e_P, A) such that (62) is satisfied for some $b^* \in [0, 1]$ but is violated for $b^* + \epsilon : \epsilon \rightarrow 0$. For all other sets of parameters the upper bound of b may be insufficient to satisfy (62).*

Proof. The RHS of (62) does not vary with b . Conversely, the LHS increases monotonically in b . An example set of such parameters is $a_H = 1, a_L = 0.5, m = 1, k = 2, e_P = 1$, and $A = 0.3$, which yields $b^* \simeq 0.2521$. ■

According to proposition 6, at the critical value b^* , H will want to switch type from H to H' . It is worth noticing that for $b < b^*$, relative performance compensation decreases both e_L and e_H . On the contrary, it increases the quantity of cheating transacted, $e_H - e_L$, because L responds to the implementation of low-powered relative evaluation by substituting effort with cheating. Hence, for values of b below the critical value, relative performance evaluation has the same devastating results on the performance of agents that we previously derived in our baseline model.

9. Conclusion

We develop an agency model with risky information transmission between agents in order to investigate the incentives and the effects of cheating. We consider two risk neutral agents who interact, coordinate but also choose private strategies with respect to exerting costly effort and supplying or requesting information. Without assuming any synergy effects in the production functions of the agents, neither allowing for any covariance of their outputs, we are able to show that the two agents experience asymmetrical risks and benefits from cheating, which result in strategic behaviors and create horizontal externalities.

We find that the mere potential of cheating creates strategic effects. Specifically, in anticipation of a risky information exchange, a higher ability agent will moderate her effort in order to limit the available information to be transmitted to a lower ability agent, and therefore cap her risk. Our model also shows that a relative compensation scheme may cause cheating activity to surge, when the coefficient of relative incentives is not sufficiently high. Moreover, even when relative incentives become powerful enough to limit cheating, this comes at a cost. The higher ability agent is vastly discouraged from exerting effort

creating serious welfare losses for the group. This result prevails even when the model is generalized to accommodate agents who are able to choose their cheating supply.

We also investigate the optimal mix of agents with respect to their abilities. We conclude that the educational output of the group is higher when similar ability agents are matched together. This is explained from the fact that as the differential of abilities between agents increases, the higher ability agent's effort increasingly deviates from the non-cheating effort. On the other hand, even though it is possible for the lower ability agent to converge towards her non-cheating effort, cheating always causes her to partially substitute effort with cheating. Furthermore, we show that the only cost-free way to deter cheating is to match agents with very similar abilities together, so that the risk of transmitting information will offset its benefit.

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